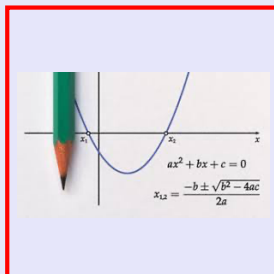


Math 125
Spring 2022
Lecture 19



More radicals:

Assume All Variables are non-negative.

Simplify

$$1) \sqrt{x^2} = \boxed{x}$$

$$4) \sqrt[3]{x^3} = \boxed{x}$$

$$2) \sqrt{25x^2} = \boxed{5x}$$

$$5) \sqrt[3]{8x^3} = \boxed{2x}$$

$$3) \sqrt{75x^3} = \sqrt{25x^2} \sqrt{3x} = \boxed{5x\sqrt{3x}}$$

$$6) \sqrt[3]{24x^4} \\ = \sqrt[3]{8x^3} \sqrt[3]{3x} \\ = \boxed{2x\sqrt[3]{3x}}$$

$$7) \sqrt[4]{x^4} = \boxed{x}$$

$$8) \sqrt[4]{16x^4} = \sqrt[4]{(2x)^4} = \boxed{2x}$$

$$9) \sqrt[4]{80x^7} = \sqrt[4]{16x^4} \sqrt[4]{5x^3}$$

$$= \boxed{2x \sqrt[4]{5x^3}}$$

$$f(x) = \sqrt[3]{x-2}$$

$$g(x) = \sqrt[3]{8x-8}$$

find

$$1) f(-25) = \sqrt[3]{-25-2}$$

$$= \sqrt[3]{-27} = \boxed{-3}$$

$$(-3)^3 = -27$$

$$2) f(127) = \sqrt[3]{127-2}$$

$$= \sqrt[3]{125} = \boxed{5}$$

$$5^3 = 125$$

$$3) g(0) = \sqrt[3]{8(0)-8}$$

$$= \sqrt[3]{0-8}$$

$$= \sqrt[3]{-8} = \boxed{-2}$$

$$4) g(-7) = \sqrt[3]{8(-7)-8}$$

$$= \sqrt[3]{-64} = \boxed{-4}$$

$$(-4)^3 = -64$$

Find the domain of $f(x)$

Hint:
Even index
 \Rightarrow Radicand ≥ 0

1) $f(x) = \sqrt{x-2}$
 index = 2 \rightarrow even index $\rightarrow x-2 \geq 0 \quad x \geq 2$
 Domain $\rightarrow [2, \infty)$

2) $f(x) = \sqrt[3]{x-2}$
 index = 3 \rightarrow odd index \rightarrow Domain $(-\infty, \infty)$

3) $f(x) = \sqrt{4-x}$
 even index $\rightarrow 4-x \geq 0 \rightarrow x \leq 4$
 $(-\infty, 4]$

4) $f(x) = \frac{1}{\sqrt{4-x}}$
 even index
 Denominator $\neq 0$
 $4-x > 0 \quad x < 4$
 $(-\infty, 4)$

Simplify

Hint:
 $\sqrt[n]{x^m} = x^{\frac{m}{n}}$

1) $\sqrt{2} \sqrt[3]{2} = 2^{\frac{1}{2}} \cdot 2^{\frac{1}{3}}$
 $= 2^{\frac{1}{2} + \frac{1}{3}} = 2^{\frac{5}{6}} = \sqrt[6]{2^5} = \sqrt[6]{32}$

2) $\frac{\sqrt{2}}{\sqrt[3]{2}} = \frac{2^{\frac{1}{2}}}{2^{\frac{1}{3}}} = 2^{\frac{1}{2} - \frac{1}{3}} = 2^{\frac{1}{6}} = \sqrt[6]{2^1} = \sqrt[6]{2}$

3) $\sqrt[3]{\sqrt{2}} = \left[2^{\frac{1}{2}}\right]^{\frac{1}{3}} = 2^{\frac{1}{2} \cdot \frac{1}{3}} = 2^{\frac{1}{6}} = \sqrt[6]{2}$

4) $\frac{\sqrt[3]{y^2}}{\sqrt[6]{y}} = \frac{y^{\frac{2}{3}}}{y^{\frac{1}{6}}} = y^{\frac{2}{3} - \frac{1}{6}} = y^{\frac{1}{2}} = \sqrt{y}$
 $\frac{2 \cdot 2}{3 \cdot 2} - \frac{1}{6} = \frac{4}{6} - \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

5) $\sqrt[5]{y \sqrt[3]{x^2}}$
 $= \left[\left(x^{\frac{2}{3}} \right)^{\frac{1}{3}} \right]^{\frac{1}{5}} = x^{\frac{2}{3} \cdot \frac{1}{3} \cdot \frac{1}{5}} = x^{\frac{2}{45}} = \sqrt[45]{x^2}$
 $\frac{2 \cdot 2}{3 \cdot 2} - \frac{1}{6} = \frac{4}{6} - \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

Multiply and Simplify:

$$1) \sqrt{3} \cdot \sqrt{7} = \sqrt{3 \cdot 7} \\ = \boxed{\sqrt{21}}$$

$$4) \sqrt[3]{54} = \sqrt[3]{27} \sqrt[3]{2} \\ = \boxed{3\sqrt[3]{2}}$$

$$2) \sqrt{2} \sqrt{10} = \sqrt{2 \cdot 10} \\ = \sqrt{20} \\ = \sqrt{4} \sqrt{5} = \boxed{2\sqrt{5}}$$

$$5) \sqrt[5]{-64} = \sqrt[5]{-32} \sqrt[5]{2} \\ = \boxed{-2\sqrt[5]{2}}$$

$$3) \sqrt{75} \\ = \sqrt{25} \sqrt{3} \\ = \boxed{5\sqrt{3}}$$

$$32 = 2^5$$

$$4) \sqrt[4]{32} \\ = \sqrt[4]{2^4} \sqrt[4]{2} \\ = \boxed{2\sqrt[4]{2}}$$

Simplify

$$\sqrt{x^5 y^{13}} = \sqrt{x^4 y^{12}} \sqrt{x y}$$

$$5 = 4 + 1 \\ 13 = 12 + 1$$

$$= \boxed{x^2 y^6 \sqrt{xy}}$$

$$\frac{4}{2} = 2$$

$$\frac{12}{2} = 6$$

Simplify: $\sqrt[3]{x^5 y^{10} z^{15}} = \sqrt[3]{x^3 y^9 z^{15}} \sqrt[3]{x^2 y}$

$$5 = 3 + 2$$

$$10 = 9 + 1$$

$$= \boxed{x y^3 z^5 \sqrt[3]{x^2 y}}$$

15 is divisible by 3.

$$\frac{3}{3} = 1, \frac{9}{3} = 3, \frac{15}{3} = 5$$

Simplify $\sqrt[5]{64 x^3 y^7 z^{29}} =$

$64 = 2^6$ $\sqrt[5]{2^6 x^3 y^7 z^{29}} =$

$\sqrt[5]{2^5 y^5 z^{25}} \sqrt[5]{2 x^3 y^2 z^4}$

$6 = 5 + 1$

3 is less than 5

$7 = 5 + 2$

$29 = 25 + 4$

Simplify $\sqrt[6]{x^5 y^7 z^{19} t^{30}}$

$= \sqrt[6]{y^6 z^{18} t^{30}} \sqrt[6]{x^5 y z}$

$= y z^3 t^5 \sqrt[6]{x^5 y z}$

Simplify

1) $\sqrt{5x} \sqrt{10x} = \sqrt{50x^2} = \sqrt{25x^2} \sqrt{2} = \boxed{5x\sqrt{2}}$

2) $\sqrt[3]{4x^2} \sqrt[3]{-2x} = \sqrt[3]{-8x^3} = \boxed{-2x}$

3) $2\sqrt{5} \cdot 3\sqrt{40} = 6\sqrt{200} = 6\sqrt{100} \sqrt{2} = 6 \cdot 10\sqrt{2} = \boxed{60\sqrt{2}}$

4) $\sqrt[5]{8x^4 y^6 z^2} \cdot \sqrt[5]{-4xy^4 z^3}$

$= \sqrt[5]{-32} x^5 y^{10} z^5 = \sqrt[5]{(-2)^5} x y z$

$-32 = (-2)^5$

$= \boxed{-2xy^2z}$

Simplify

$$1) 12\sqrt{2} - 8\sqrt{2} = (12-8)\sqrt{2} = \boxed{4\sqrt{2}}$$

$$\begin{aligned} 2) 7\sqrt{18} + 5\sqrt{8} \\ &= 7\sqrt{9}\sqrt{2} + 5\sqrt{4}\sqrt{2} \\ &= 7 \cdot 3\sqrt{2} + 5 \cdot 2\sqrt{2} = 21\sqrt{2} + 10\sqrt{2} \\ &= \boxed{31\sqrt{2}} \end{aligned}$$

$$\begin{aligned} 3) -3\sqrt[3]{2x} - 2\sqrt[3]{2x} \\ &= (-3-2)\sqrt[3]{2x} = \boxed{-5\sqrt[3]{2x}} \end{aligned}$$

$$\begin{aligned} 4) 3\sqrt{12x} - 6\sqrt{27x} \\ &= 3\sqrt{4}\sqrt{3x} - 6\sqrt{9}\sqrt{3x} = 3 \cdot 2\sqrt{3x} - 6 \cdot 3\sqrt{3x} \\ &= 6\sqrt{3x} - 18\sqrt{3x} \\ &= \boxed{-12\sqrt{3x}} \end{aligned}$$

Simplify

$$1) \sqrt[3]{x} (\sqrt[3]{2} - \sqrt[3]{x^2}) = \sqrt[3]{2x} - \sqrt[3]{x^3} = \boxed{\sqrt[3]{2x} - x}$$

$$\begin{aligned} 2) -2\sqrt{3}(5\sqrt{3} + 3\sqrt{2}) \\ &= -10\sqrt{9} - 6\sqrt{6} = -10 \cdot 3 - 6\sqrt{6} = \boxed{-30 - 6\sqrt{6}} \end{aligned}$$

$$\begin{aligned} 3) (\sqrt{7} + \sqrt{3})(\sqrt{7} - \sqrt{3}) \\ &= \sqrt{49} - \sqrt{21} + \sqrt{21} - \sqrt{9} = 7 - 3 = \boxed{4} \end{aligned}$$

$$\begin{aligned} 4) (2\sqrt{3} + \sqrt{2})^2 \\ &= (2\sqrt{3} + \sqrt{2})(2\sqrt{3} + \sqrt{2}) \\ &= 4\sqrt{9} + 2\sqrt{6} + 2\sqrt{6} + \sqrt{4} \\ &= 4 \cdot 3 + 4\sqrt{6} + 2 \\ &= \boxed{14 + 4\sqrt{6}} \end{aligned}$$

Hint:
 $x^2 = x \cdot x$

Simplify

$$\begin{aligned}
 1) (\sqrt{6} - \sqrt{3})^2 &= (\sqrt{6} - \sqrt{3})(\sqrt{6} - \sqrt{3}) \\
 &= \sqrt{36} - \sqrt{18} - \sqrt{18} + \sqrt{9} \\
 &= 6 - 2\sqrt{18} + 3 \\
 &= 9 - 2\sqrt{9}\sqrt{2} = \boxed{9 - 6\sqrt{2}}
 \end{aligned}$$

$$\begin{aligned}
 2) (\sqrt[3]{2} - 1)(\sqrt[3]{4} + \sqrt[3]{2} + 1) \\
 = \sqrt[3]{8} + \cancel{\sqrt[3]{4}} + \cancel{\sqrt[3]{2}} - \cancel{\sqrt[3]{4}} - \cancel{\sqrt[3]{2}} - 1 = 2 - 1 = \boxed{1}
 \end{aligned}$$

$$\begin{aligned}
 3) (\sqrt[3]{4} + \sqrt[3]{3})(\sqrt[3]{16} - \sqrt[3]{12} + \sqrt[3]{9}) \\
 = \sqrt[3]{64} - \cancel{\sqrt[3]{48}} + \cancel{\sqrt[3]{36}} + \cancel{\sqrt[3]{48}} - \cancel{\sqrt[3]{36}} + \sqrt[3]{27} \\
 = 4 + 3 = \boxed{7}
 \end{aligned}$$

Solve & check:

$$1) \sqrt{2x+1} = 5$$

Index = 2

$$(\sqrt{2x+1})^2 = 5^2$$

$$2x+1 = 25$$

$$2x = 24$$

$$\boxed{x = 12}$$

$$\{12\}$$

check:

$$\sqrt{2(12)+1} = 5$$

$$\sqrt{25} = 5$$

$$5 = 5\checkmark$$

$$2) \sqrt[3]{2x+1} = -5$$

Index = 3

$$(\sqrt[3]{2x+1})^3 = (-5)^3$$

$$2x+1 = -125$$

$$\rightarrow 2x = -126$$

$$\boxed{x = -63}$$

$$\{-63\}$$

check

$$\sqrt[3]{2(-63)+1} = -5$$

$$\sqrt[3]{-126+1} = -5$$

$$\sqrt[3]{-125} = -5$$

$$\sqrt{-5} = -5$$

Solve $x + \sqrt{26 - 11x} = 4$

$$\sqrt{26 - 11x} = 4 - x$$

$$(\sqrt{26 - 11x})^2 = (4 - x)^2$$

$$26 - 11x = (4 - x)(4 - x)$$

$$26 - 11x = 16 - 4x - 4x + x^2$$

$$26 - 11x = 16 - 8x + x^2$$

$$0 = 16 - 8x + x^2 - 26 + 11x$$

$$\rightarrow 0 = x^2 + 3x - 10$$

$$0 = (x + 5)(x - 2)$$

$$x + 5 = 0 \rightarrow x = -5$$

$$x - 2 = 0 \rightarrow x = 2$$

we must check every solution in the original problem.

Check $x = -5$

$$x + \sqrt{26 - 11x} = 4$$

$$-5 + \sqrt{26 - 11(-5)} = 4$$

$$-5 + \sqrt{26 + 55} = 4$$

$$-5 + \sqrt{81} = 4$$

$$-5 + 9 = 4$$

$$4 = 4 \checkmark$$

Check $x = 2$

$$x + \sqrt{26 - 11x} = 4$$

$$2 + \sqrt{26 - 11(2)} = 4$$

$$2 + \sqrt{26 - 22} = 4$$

$$2 + \sqrt{4} = 4$$

$$2 + 2 = 4$$

$$4 = 4 \checkmark$$

$\{-5, 2\}$

Solve and check:

$$2\sqrt{x-3} + 4 = x + 1$$

$$2\sqrt{x-3} = x + 1 - 4$$

$$2\sqrt{x-3} = x - 3$$

Square both Sides

$$(2\sqrt{x-3})^2 = (x-3)^2$$

$$4(x-3) = (x-3)(x-3)$$

$$4x - 12 = x^2 - 3x - 3x + 9$$

$$4x - 12 = x^2 - 6x + 9$$

$$0 = x^2 - 6x + 9 - 4x + 12$$

$$0 = x^2 - 10x + 21$$

$$0 = (x-3)(x-7)$$

$$x-3=0 \quad x-7=0$$

$$x=3 \quad x=7$$

Check:

$$2\sqrt{3-3} + 4 = 3 + 1$$

$$2 \cdot 0 + 4 = 4$$

$$0 + 4 = 4 \checkmark$$

$$2\sqrt{7-3} + 4 = 7 + 1$$

$$2\sqrt{4} + 4 = 8$$

$$2 \cdot 2 + 4 = 8 \checkmark$$

$\{3, 7\}$

Solve & check:

$$(2x+3)^{1/4} + 7 = 9$$

$$\sqrt[4]{2x+3} + 7 = 9$$

$$\sqrt[4]{2x+3} = 9 - 7$$

$$\sqrt[4]{2x+3} = 2$$

index = 4

$$(\sqrt[4]{2x+3})^4 = (2)^4$$

$$2x+3 = 16$$

$$2x = 13$$

$$x = \frac{13}{2}$$

check

$$\sqrt[4]{2(\frac{13}{2})+3} + 7 = 9$$

$$\sqrt[4]{13+3} + 7 = 9$$

$$\sqrt[4]{16} + 7 = 9$$

$$2+7=9 \checkmark$$

$\{\frac{13}{2}\}$

Solve and check:

$$2\sqrt{4x+1} - 9 = x - 5$$

$$2\sqrt{4x+1} = x - 5 + 9$$

$$2\sqrt{4x+1} = x + 4$$

$$(2\sqrt{4x+1})^2 = (x+4)^2$$

$$4(4x+1) = x^2 + 8x + 16$$

$$16x + 4 = x^2 + 8x + 16$$

$$\{2, 6\}$$

$$\rightarrow 0 = x^2 + 8x + 16 - 16x - 4$$

$$0 = x^2 - 8x + 12$$

$$0 = (x-2)(x-6)$$

$$x-2=0 \quad x-6=0$$

$$\boxed{x=2} \quad \boxed{x=6}$$

check $x=2$

$$2\sqrt{4(2)+1} - 9 = 2 - 5$$

$$2 \cdot 3 - 9 = -3$$

$$-3 = -3 \checkmark$$

check $x=6$

$$2\sqrt{4(6)+1} - 9 = 6 - 5$$

$$2\sqrt{25} - 9 = 1$$

$$10 - 9 = 1 \checkmark$$